

# ANFIS Approach for Tracking Control of MEMS Triaxial Gyroscope

Mohsen Rakhshan, Mokhtar Shasadeghi, and Faridoon Shabani-nia

## Abstract

In this paper, an Adaptive Neuro Fuzzy Inference System (ANFIS) based control is proposed for the tracking of a Micro-Electro Mechanical Systems (MEMS) gyroscope sensor. The ANFIS is used to train parameters of the controller for tracking a desired trajectory. Numerical simulations for a MEMS gyroscope are looked into to check the effectiveness of the ANFIS control scheme. It proves that the system using the proposed ANFIS controller has better tracking performance than that using only a fuzzy control or Neural network approach to control over the existence of external perturbations.

**Keywords**— ANFIS, Gyroscope, MEMS, Tracking

## I. INTRODUCTION

MEMS gyroscopes have a broad application scope of the robotics, automotive and consumer-electronics markets. This is by reason of their reduced costs, size and integration aptitude. A gyroscope is a regularly used sensor for measuring angular velocity in many fields of application, such as navigation, motion control. The performance of the MEMS gyroscope is affected by time-varying parameters as well as noise sources, quadrature errors, parameter variations and external disturbances, which cause a frequency of oscillation mismatch between the two vibrating axes [1].

Advanced control approaches, such as intelligent control, are the effective methods for controlling MEMS gyroscopes.

In the last years, several control approaches have been shown to control the MEMS gyroscope. Batur [2] build up a sliding mode control for a MEMS gyroscope. Park in [3] presented an adaptive controller of a MEMS gyroscope. Fei in [1], [4]–[6] developed an adaptive sliding mode controller and a robust adaptive controller for a MEMS gyroscope. Recently much research has been done to apply intelligent control approaches such as neural nets and fuzzy

controls. Fei in [5] developed an adaptive Fuzzy control approach and in [1] he applied a robust adaptive neural sliding mode approach on a MEMS gyroscope.

In this paper, we align with the design of an ANFIS control. By applying ANFIS approach with considering system uncertainties. The proposed method is developed by combining feed forward techniques and ANFIS properties. We use this method to obligate the gyroscope to follow any arbitrary trajectory. In comparison to the past studies, many of them uses the sliding mode control besides the fuzzy control or neural network control, but because of the existence of the disturbance, system have chattering and the time to reach to the steady state is not desirable. In this proposed method, the response of the system is fast and almost has no chattering.

## II. DYNAMICS OF THE MEMS GYROSCOPE

The dynamics of the MEMS gyroscope are shown in Figure 1. A typical MEMS gyroscope structure includes a proof mass suspended from spring beams, sensing mechanisms for forcing an oscillatory motion, electrostatic actuations and sensing the position and velocity of the proof mass, also a rigid frame which is rotated around the rotation axis. Newton's law in the rotating frame determines the dynamics of a MEMS gyroscope [4].

Respect to an inertial system of reference the gyroscope is moving with a constant linear speed; also, the gyroscope is rotating at a constant angular velocity; and the centrifugal forces are accepted to be negligible. The nonlinear motion equations of such a Triaxial gyroscope that rotates on the  $x$ ,  $y$  and  $z$  axis can be derived as [1]:

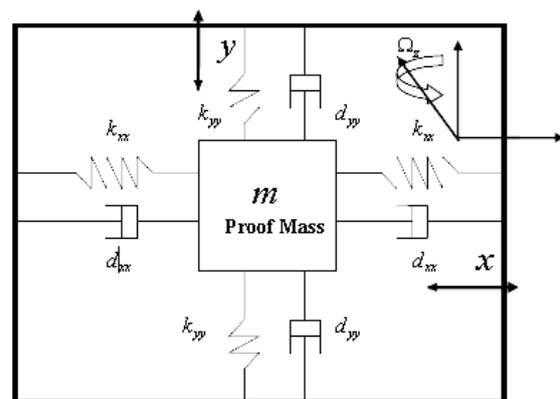


Fig. 1. Simplified model of a z-axis MEMS gyroscope [6].

Manuscript received May 18, 2014; revised October 18, 2014; accepted October 21, 2014.

M. Rakhshan, Department of Electrical Engineering, Shiraz University of Technology, Shiraz, Iran (e-mail: m.rakhshan@sutech.ac.ir).

M. Shasadeghi, Department of Electrical Engineering, Shiraz University of Technology, Shiraz, Iran (e-mail: shasadeghi@sutech.ac.ir).

F. Shabani-nia, Department of Electrical Engineering, University of Shiraz, Shiraz, Iran (e-mail: shabani@shirazu.ac.ir).

$$\begin{aligned}
m\ddot{x} + d_{xx}\dot{x} + d_{yy}\dot{y} + d_{zz}\dot{z} + k_{xx}x + k_{yy}y + k_{zz}z - m(\Omega_y^2 + \Omega_z^2)x + m\Omega_x\Omega_y y + m\Omega_x\Omega_z z = u_x + 2m\Omega_z\dot{y} - 2m\Omega_y\dot{z} \\
m\ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + d_{yz}\dot{z} + k_{xy}x + k_{yy}y + k_{yz}z - m(\Omega_x^2 + \Omega_z^2)y + m\Omega_x\Omega_y x + m\Omega_y\Omega_z z = u_y - 2m\Omega_z\dot{x} + 2m\Omega_x\dot{z} \\
m\ddot{z} + d_{xz}\dot{x} + d_{yz}\dot{y} + d_{zz}\dot{z} + k_{xz}x + k_{yz}y + k_{zz}z - m(\Omega_x^2 + \Omega_y^2)z + m\Omega_x\Omega_z x + m\Omega_y\Omega_z y = u_z + 2m\Omega_x\dot{y} - 2m\Omega_y\dot{x}
\end{aligned} \quad (1)$$

where  $m$  is the mass of proof mass fabrication faults inducing essentially from the asymmetric spring terms  $k_{xy}$ ,  $k_{xz}$  and  $k_{yz}$  and asymmetric damping terms  $d_{xy}$ ,  $d_{yz}$  and  $d_{xz}$ ; Spring terms are  $k_{xx}$ ,  $k_{yy}$  and  $k_{zz}$ ; Damping terms are  $d_{xx}$ ,  $d_{yy}$  and  $d_{zz}$ ; Angular velocities are  $\Omega_x$ ,  $\Omega_y$  and  $\Omega_z$  and finally  $u_x$ ,  $u_y$  and  $u_z$  are the control forces in the  $x$ ,  $y$  and  $z$  directions consequently [1].

Dividing the equation (1) by the reference mass, and with considering about the non-dimensional time  $t^* = \omega_0 t$ , and after that with dividing two sides of the equation by the reference frequency  $\omega_0^2$  and also the reference length  $q_0$  we will rewrite the dynamic equation in vector form:

$$\ddot{q} + \frac{D}{m\omega_0^2 q_0} \dot{q} + \frac{k_a}{m\omega_0^2 q_0} q + \frac{\Omega_b}{m\omega_0^2 q_0} q = \frac{u}{m\omega_0^2 q_0} - 2 \frac{\Omega}{\omega_0} \dot{q} \quad (2)$$

where

$$q = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix}, \\
u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}, \quad D = \begin{bmatrix} d_{xx} & d_{xy} & d_{xz} \\ d_{xy} & d_{yy} & d_{yz} \\ d_{xz} & d_{yz} & d_{zz} \end{bmatrix}, \\
k_a = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{xy} & k_{yy} & k_{yz} \\ k_{xz} & k_{yz} & k_{zz} \end{bmatrix}, \\
\Omega_b = \begin{bmatrix} -(\Omega_y^2 + \Omega_z^2) & \Omega_x\Omega_y & \Omega_x\Omega_z \\ -(\Omega_x^2 + \Omega_z^2) & \Omega_y\Omega_x & \Omega_y\Omega_z \\ -(\Omega_x^2 + \Omega_y^2) & \Omega_x\Omega_z & \Omega_y\Omega_z \end{bmatrix}.$$

We define the new parameters as follows:

$$q^* = \frac{q}{q_0}, \quad D^* = \frac{D}{m\omega_0^2}, \quad \Omega^* = \frac{\Omega}{\omega_0}, \quad u_x^* = \frac{u_x}{m\omega_0^2 q_0}, \\
u_y^* = \frac{u_y}{m\omega_0^2 q_0}, \quad u_z^* = \frac{u_z}{m\omega_0^2 q_0}, \quad \omega_x = \sqrt{\frac{k_{xx}}{m\omega_0^2}},$$

$$\omega_y = \sqrt{\frac{k_{yy}}{m\omega_0^2}}, \quad \omega_z = \sqrt{\frac{k_{zz}}{m\omega_0^2}}, \quad \omega_{xy} = \frac{k_{xy}}{m\omega_0^2}, \\
\omega_{yz} = \frac{k_{yz}}{m\omega_0^2}, \quad \omega_{xz} = \frac{k_{xz}}{m\omega_0^2}.$$

With these definitions, we achieve to the ultimate form of the non-dimensional equation [1]:

$$\ddot{q} + Dq + k_b q + \Omega_b q = u - 2\Omega q \quad (3)$$

where

$$k_b = \begin{bmatrix} \omega_x^2 & \omega_{xy} & \omega_{xz} \\ \omega_{xy} & \omega_y^2 & \omega_{yz} \\ \omega_{xz} & \omega_{yz} & \omega_z^2 \end{bmatrix}.$$

### III. NEUROFUZZY MODELING

We use a neural network to model a dynamic plant by a nonlinear regression in the discrete time domain. The output is a network, with adapted weights, which resembles the plant. It is an issue that the learning results in a broad set of parameter values which makes it almost impossible to clarify in words. Contrarily, a fuzzy rule base consists of an if-then statement that is approximately our lingual words and are readable, Again, the problem here is that it cannot learn the rules by itself; so the solution is to combine neural network with fuzzy logic in neurofuzzy systems in order to achieve readability and learning ability concurrently [7]. The rules usually are set up by a human expert and the neurofuzzy controller has to learn and achieve an optimal rule [8]. The main part of neurofuzzy cooperation comes from a common scheme called adaptive networks, which consolidate both neural networks and fuzzy models. The fuzzy model within the scheme of adaptive networks is called ANFIS, which acquire certain advantages over neural networks [9]. ANFIS, which can provide a set of fuzzy if-then statements as the rules with suitable tuning of membership functions to generate the customary input output data pairs. It can take on a greatly nonlinear mapping. Therefore, it is well suited for nonlinear dynamics. ANFIS consists of fuzzy rules, which are absolutely local mappings that after the adaptation should reduce the output error for the current training practice and also minimize disturbances to the response already learned. ANFIS requires defining membership functions and fuzzy rules before the training. Here, the goal is to attain an appropriate control action with the ANFIS model. Membership functions and rules affect the output of the control network, which is used to minimize error with adapting the structure and/or membership function parameters of the neurofuzzy control network. In this current usage, the structure of the fuzzy control network is kept constant and only

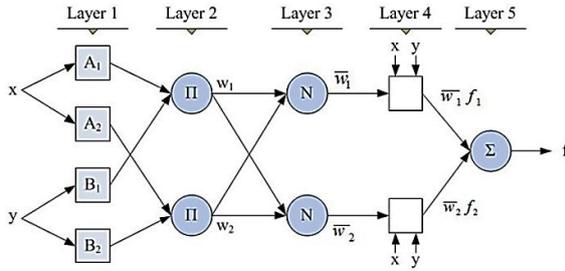


Fig. 2. The structure of the ANFIS network.

membership function parameters are set to deliver the desired control action [10].

Fig. 2 demonstrates the basic 2-input 1-output structure of the ANFIS network for a first order Sugeno fuzzy system. The Layer-1 lies in membership functions depicted by the generalized bell function [11]:

$$\mu(X) = (1 + ((X - c) / a)^{2b})^{-1} \quad (4)$$

Where adaptable parameters are  $a$ ,  $b$  and  $c$ . Layer-2 applies the fuzzy intersection operator, and Layer-3 regulates the firing strengths. The output after crossing the Layer-4 consists of a linear aggregation of the firing strength  $w$  which is normalized multiplied by inputs [10]:

$$Y = w (pX + r) \quad (5)$$

Where adaptable parameters are  $p$  and  $r$ . Layer-5 acts as a summation on the outputs of Layer-4. The adoption of modifiable parameters has two steps. First, data are propagated forward in the network structure until it reaches to Layer-4 where the parameters are identified by a least-squares estimator (LMS) [12]. Then the parameters in Layer-2 are adapted using gradient descent.

Back propagation is the method which ANFIS uses to determine the parameters of membership functions and least mean square estimation is the means to modify the consequent parameters.

The designer should perform the following steps for using the ANFIS:

1. Design a Sugeno Fuzzy inference system (FIS) according to conditions of the problem.
2. Optimize the FIS with actual input data.
3. Set up training and testing data in matrices which determine the inputs and outputs.
4. Train the FIS using training data with ANFIS algorithm.
5. Test the trained system using the testing data [11].

#### IV. ANFIS BASED CONTROLLER

In this section, an ANFIS based controller can be designed for the MEMS gyroscope with unknown system nonlinearities. The proposed controller is basically a fuzzy controller that uses the tracking error to change adaptively using neural network training

algorithms. The tracking error has classified in 7 classes: Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (ZE), Positive Small (PS), Positive Medium (PM) and Positive Big (PB). The second output for the fuzzy system is derivative of the tracking error. This input is classified as the first input. The output of the Sugeno FIS is Negative, Zero, Positive. The rule base of the FIS is a rule base which is used for a Fuzzy PI controller which is omitted for brevity. After defining the fuzzy controller, the real data are used to train the FIS using ANFIS algorithms.

Consider the dynamics with parametric uncertainties and external disturbance as:

$$\dot{q} + (D + 2\Omega + \Delta D)\dot{q} + (k_b + \Delta k_b)q + \Omega_b q = u - d \quad (6)$$

where  $\Delta D$  is the unknown parameter uncertainty of the matrix,  $D + 2\Omega$ ,  $\Delta k_b$  is the unknown parameter uncertainty of the matrix  $k_b$  and  $d$  is the external disturbance of the system. Rewriting (6) as:

$$\dot{q} + (D + 2\Omega)\dot{q} + k_b q = u + d_1 \quad (7)$$

where  $d_1$  expresses the lumped model uncertainties and external disturbances which are given by  $d_1 = d - \Delta D\dot{q} - \Delta k_b q$ .

Rewriting (7) as:

$$\ddot{q} = -(D + 2\Omega)\dot{q} - k_b q + \Omega_b q + u + d_1 \quad (8)$$

Define [1]:

$$f(q, \dot{q}, t) = -(D + 2\Omega)\dot{q} - k_b q - \Omega_b q + d_1 \quad (9)$$

where  $f(q, \dot{q}, t)$  is an unknown nonlinear function. Therefore, (8) becomes:

$$\ddot{q} = f(q, \dot{q}, t) + u \quad (10)$$

The control goal for the MEMS gyroscope is to obligate the proof mass to oscillate in the  $x$ ,  $y$  and  $z$  directions at a desired frequency and amplitude  $x_m = A_1 \sin(\omega_1 t)$ ,  $y_m = A_2 \sin(\omega_2 t)$ , and  $z_m = A_3 \sin(\omega_3 t)$ .

Then, the reference model can be defined as:

$$\ddot{q}_m + k_m q_m = 0 \quad (11)$$

where

$$q_m = [x_m \ y_m \ z_m]^T, \quad k_m = \text{diag} \{ \omega_1^2, \omega_2^2, \omega_3^2 \}.$$

Define the tracking error as follows [1]:

$$e = q - q_m \quad (12)$$

The block diagram of the control system is depicted in Fig. 3. As it is shown in the Fig. 3 the input of the ANFIS controller is the tracking error and also the ANFIS adaptive law and the end product of the ANFIS controller is  $u$  which directly effects on the MEMS gyroscope.

### V. A Simulation Study

In this section, we will assess the considered ANFIS approach on the lumped MEMS gyroscope sensor model. The parameters of the MEMS gyroscope sensor are as follows [1]:

$$\begin{aligned} m &= 0.57e-8 \text{ kg} , \quad \omega_0 = 1 \text{ kHz} , \quad q_0 = 10^{-6} \text{ m} , \\ d_{xx} &= 0.429e-6 \text{ N s / m} , \\ d_{yy} &= 0.0429e-6 \text{ N s / m} , \\ d_{zz} &= 0.895e-6 \text{ N s / m} , \\ d_{xy} &= 0.0429e-6 \text{ N s / m} , \\ d_{xz} &= 0.0687e-6 \text{ N s / m} , \\ d_{yz} &= 0.0895e-6 \text{ N s / m} , \quad k_{xx} = 80.98 \text{ N / m} , \\ k_{xy} &= 5 \text{ N / m} , k_{yy} = 71.62 \text{ N / m} , \\ k_{zz} &= 71.62 \text{ N / m} , \quad k_{xz} = 6 \text{ N / m} , \\ k_{yz} &= 7 \text{ N / m} . \end{aligned}$$

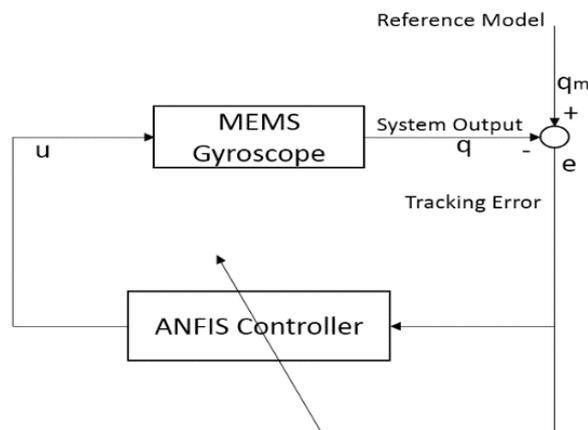


Fig. 3. The block diagram of the control system

Since the general displacement range of the MEMS gyroscope sensor in each axis is at the sub-micrometer level, it is acceptable to choose  $1 \mu\text{m}$  as the reference length  $q_0$ . Because the usual natural frequency of each axis of a vibratory MEMS gyroscope sensor is in the KHz level,  $\omega_0$  is chosen as 1 kHz. The unknown angular velocity is assumed to be  $\Omega_z = 5.0 \text{ rad / s}$ ,  $\Omega_x = 3.0 \text{ rad / s}$  and  $\Omega_y = 2.0 \text{ rad / s}$ . The desired motion trajectories are  $x_m = \sin(\omega_1 t)$ ,  $y_m = 1.2 \sin(\omega_2 t)$  and  $z_m = 1.5 \sin(\omega_3 t)$  where  $\omega_1 = 6.71 \text{ kHz}$ ,  $\omega_2 = 5.11 \text{ kHz}$ ,  $\omega_3 = 4.17 \text{ kHz}$ . The initial values of the rules weight in the ANFIS network is assumed to be  $\omega_0 = [0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1]^T$ . The initial states of the MEMS dynamics are  $[0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$ . Notice that we calculated the disturbances (external disturbances is  $d(t) = 100 \sin(2\pi t)$  in this case) in the unknown  $f(q, \dot{q}, t)$  function. After figuring out the equation figures 4-6 will be generated. Figures 7-8 depicts the position tracking error of the  $x$  and  $y$  directions using testing data of ANFIS controller. It is clear to the nonlinear differential equation that we need to solve it Rewriting the equation 5 with the real parameters, we access using numerical methods. From figures 7-8 it is clear that the trained motions follow the desired motion in just one training process with maximum error of 0.3. Put differently, the MEMS gyroscope can maintain the proof mass to vibrate in the  $x$ ,  $y$  and  $z$  directions at a given frequency and amplitude by using the ANFIS control. The advantage of the proposed ANFIS controller is that it does not depend on accurate mathematical models, which are difficult to obtain and may not yield satisfactory performance under parameter variations. The simulation results show that the system is capable of tracking the desired vibration trajectory determined by the reference model output; the execution of the ANFIS control is satisfactory in the existence of unknown system nonlinearities.

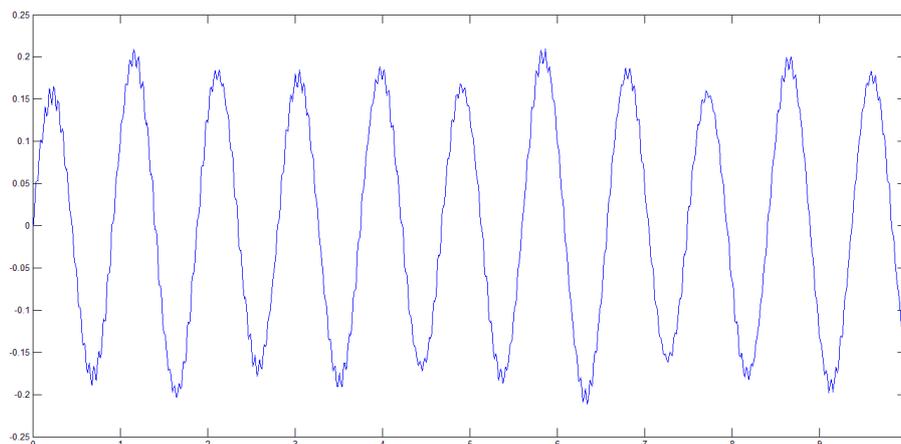


Fig. 4. The numerical solve for equation 5 in x axis.

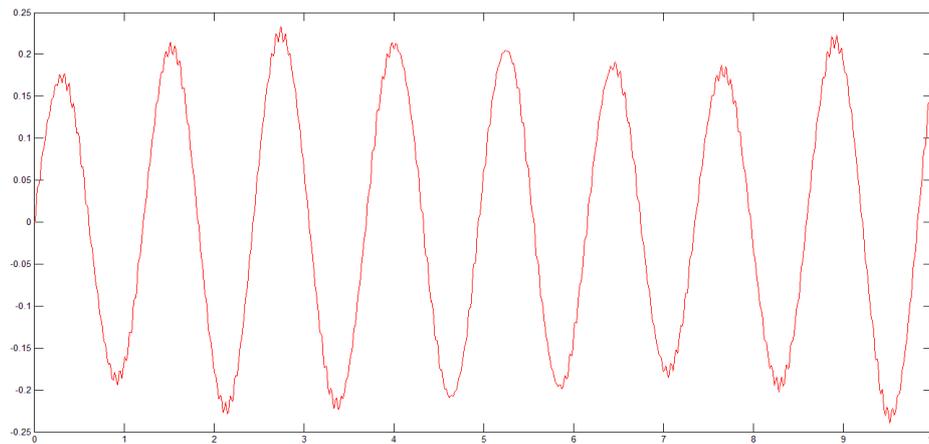


Fig. 5. The numerical solve for equation 5 in y axis.

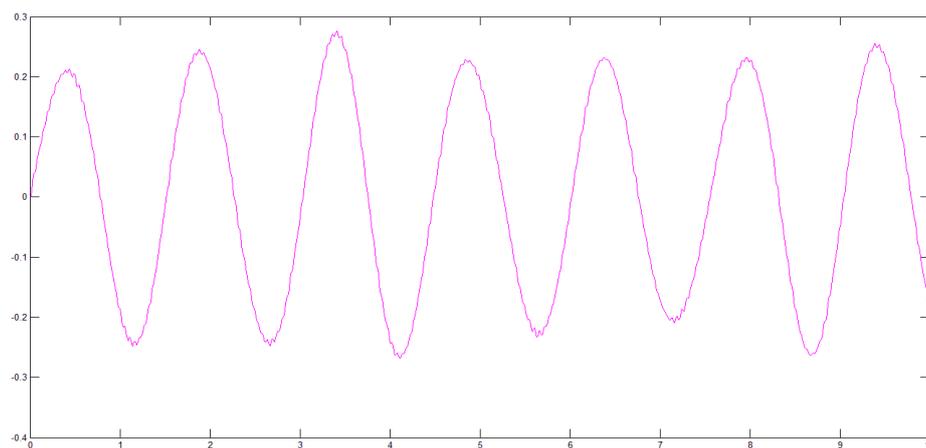


Fig. 6. The numerical solve for equation 5 in z axis.

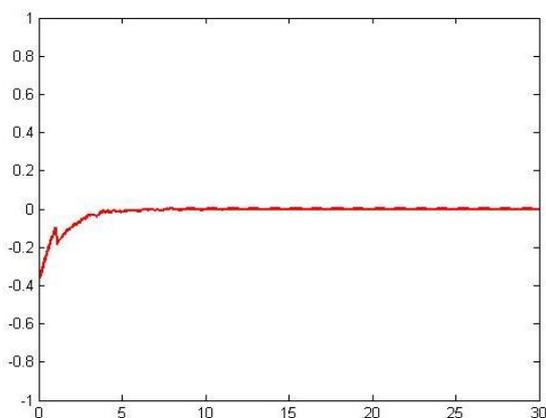


Fig. 7. Tracking error using the ANFIS control in x axis.

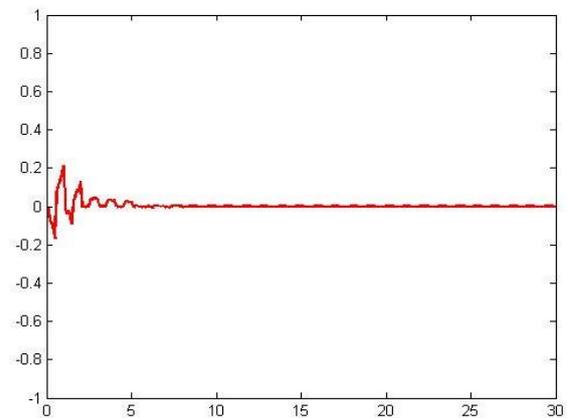


Fig. 8. Tracking error using the ANFIS control in y axis.

### VI. Conclusion

An ANFIS control approach is proposed for the Triaxial angular velocity sensor. Numerical simulation demonstrated the satisfactory performance of the proposed ANFIS control scheme in the presence of model uncertainties and external disturbances. We

could show that ANFIS try to train the input signals to follow our desired motions trajectory in a very short time. In comparison with the other studies have done before with neural network or fuzzy control for the MEMS gyroscope that is provided in the referenced papers and other researches, ANFIS has better and faster performance.

## REFERENCES

- [1] J. Fei, H. Ding, H. Shixi, W. Shitao, and M. Xin, "Robust Adaptive Neural Sliding Mode Approach for Tracking Control of a MEMS Triaxial Gyroscope," *Int. J. Adv. Robot. Syst.*, vol. 9, pp. 1–9, 2012.
- [2] C. Batur and T. Sreeramreddy, "Sliding Mode Control of a Simulated MEMS Gyroscope," *ISA Trans.*, vol. 45, no. 1, pp. 99–108, 2006.
- [3] R. Park, R. Horowitz, S. Hong, and Y. Nam, "Trajectory Switching Algorithm for a MEMS Gyroscope," *IEEE Trans. Instrum. Meas.*, vol. 56, no. 60, pp. 2561–2569, 2007.
- [4] J. Fei and C. Batur, "Robust Adaptive Control for a MEMS Vibratory Gyroscope," *Int. J. Adv. Manuf. Technol.*, vol. 42, no. 3, pp. 293–300, 2009.
- [5] J. Fei, W. Juan, and L. Tianhua, "An Adaptive Fuzzy Control Approach for the Robust Tracking of a MEMS Gyroscope sensor," *Int. J. Adv. Manuf. Technol.*, vol. 9, no. 1, pp. 25–33, 2011.
- [6] J. Fei and C. Batur, "A Novel Adaptive Sliding Mode Control with Application to MEMS Gyroscope," *ISA Trans.*, vol. 48, no. 1, pp. 73–78, 2009.
- [7] J. Jan, "Neurofuzzy Modeling," *Электронное издание*, vol. 9, no. 1, pp. 1–28, 1998.
- [8] R. LEONID, *Fuzzy Controllers Handbook*. 1997, pp. 98–105.
- [9] J.-S. R. Jang and S. Chuen-Tsai, "Neuro-Fuzzy Modeling and Control," *Proc. IEEE*, vol. 25, no. 1, pp. 378–406, 1995.
- [10] S. R. Navghare and G. L. B. Shruti, "Design of Adaptive pH Controller using ANFIS," *Int. J. Comput. Appl.*, vol. 33, no. 6, pp. 887–896, 2011.
- [11] B. Narendra, "ANFIS based HVDC control and fault identification of HVDC converter," *HAIT J. Sci. Eng.*, vol. 2, no. 5, pp. 673–689, 2011.
- [12] H. Zoltan, G. Dévényi, J. Kovács, and U. Kortela, "Control of combustion base on neuro-fuzzy model," in *Ifac proceeding*, 2005, pp. 252–259.